

about three weeks, it must stand to reason that a considerable number may be expected to show a duration of at least five or six weeks, if not very occasionally more.

Captain Tupman has clearly pointed out the special conditions required to produce a nearly fixed meteor radiant for several weeks, viz. "The meteor orbit must nearly coincide with the plane of the ecliptic, the perihelion distance of the central position be a little less than unity, and the motion direct. The position would be  $90^\circ$  before the Sun at the middle time."

I am inclined, however, to think that another reason, which may assist in explaining the apparent difficulty of a rather frequent long duration with fixity of radiant, may be found in the supposition that the zone or ring of meteors belonging to a majority of showers may be a good deal wider or more diffuse than hitherto supposed, possessing a width in fact of several millions of miles, and this at the same time not improbably coincident with a considerable degree of parallelism in the direction of the orbits of the Earth and meteors at the time.

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*On the Solution by Trial of Lambert's Theorem, in Olbers' Method for the Computation of Parabolic Orbits.\** By R. H. M. Bosanquet, Fellow of St. John's College, Oxford.

In all the different forms of the method of Olbers for the computation of parabolic orbits there is a certain step, depending in principle on the finding by trial of values of the heliocentric radii vectores of the comet which satisfy the condition given by the application of Lambert's Theorem to the observed times.

In the Appendix to Olbers' treatise, at pp. 68-71 of Encke's edition (1864), a method is given by which, from any assumption as to the value of the chord joining the first and third places of the comet, or of the sum of the corresponding geocentric radii vectores, more accurate values can be deduced by Lambert's theorem. This discussion contains the germs of the principal subsequent improvements of the method; and the form given to Lambert's theorem in the Note (Encke's *Olbers*, p. 70) is that on which the present discussion will be based.

In the Gaussian modification of Olbers' method, a plan is commonly adopted which is in principle the same as that recommended by Olbers in the place cited. It consists in assuming a value of  $u$ , for which Encke has given the formula

$$\left\{ \left( \frac{t'' - t}{41} \right)^2 - A^2 \right\}^{\frac{1}{2}},$$

\* A paper on this subject was written by the author some years ago, and appeared in the *Astronomische Nachrichten* in 1872. On reading it over the author thinks that the method may still be worth the attention of the Society. The investigation is substantially unaltered, but the discussion of the example has been revised.

determining  $r$  and  $r''$  (heliocentric radii vectores) on this hypothesis; thence  $k^2$  (square of chord) by the auxiliary table of Encke, and then  $u$  again from the equation

$$k^2 = u^2 + A^2.$$

The value of  $u$  so obtained is generally better than that originally assumed, just as in the original rules of Olbers the value of  $k^2$  resulting was better than that originally assumed.

So far as I know, however, the conditions of this improvement have not been directly investigated; and it has seemed to me worth while to examine under what conditions and to what extent this improvement takes place. I have been thus led to the following simple method, by which it would appear that, after calculation of two hypotheses as to the value of  $u$  and of a certain formula, a value of  $u$  may be deduced nearly as accurate as five-place calculations will admit.

Let the value of  $u$  first assumed be called  $u_o$ , and let that obtained after calculation of  $r$ ,  $r''$ , and  $k^2$  be  $u_1$ . Let the true value of  $u$  be called  $u$ , and let

$$u_o = u + \delta u_o, \quad u_1 = u + \delta u_1.$$

If we then seek a relation between  $\delta u_o$  and  $\delta u_1$ , then by means of this relation and the equation

$$u_o - u_1 = \delta u_o - \delta u_1,$$

the values  $\delta u_o$ ,  $\delta u_1$ , can be determined, and a value of  $u$  deduced closely approximating to the true value required.

In fact, if we combine a result of the form

$$\delta u_1 = -F \delta u_o,$$

with

$$u_o - u_1 = \delta u_o - \delta u_1,$$

we get

$$\delta u_o - \delta u_1 = -\left(\frac{1}{F} + 1\right) \delta u_1 = u_o - u_1,$$

and therefore

$$\delta u_1 = -(u_o - u_1) \frac{F}{F + 1};$$

whence

$$\begin{aligned} u &= u_1 - \delta u_1 \\ &= u_1 + (u_o - u_1) \frac{F}{F + 1}, \end{aligned}$$

which is the expression for the improved value of  $u$ .

To deduce an expression for  $F$ :—

Putting  $u + \delta u_o$  for  $u$  in the Gaussian equation

$$r^2 = \left(\frac{u + c}{b}\right)^2 + B^2,$$

we have for the corresponding error in  $r$

$$\delta r = \delta u_o \frac{u+c}{rb^2},$$

neglecting the squares of the errors ; similarly

$$\delta r'' = \delta u_o \frac{u+c''}{r''b''^2} ;$$

and if we write  $r+r'' = S$ ,

$$\delta S = \delta u_o \left( \frac{u+c}{rb^2} + \frac{u+c''}{r''b''^2} \right). \tag{I}$$

Now if in Lambert's theorem we put

$$\mathfrak{S} = \kappa \tau, (\log \kappa = 8.5366114, \text{ Encke's } Olbers, \text{ p. 251}),$$

where  $\tau$  is the interval of time between the first and last observations, we can expand  $k^2$ , the square of the chord joining the first and last positions of the comet, by means of Lambert's theorem, in terms of  $\mathfrak{S}$  and  $S$ . Thus:—

$$k^2 = \frac{\mathfrak{S}^2}{S} + \frac{\mathfrak{S}^4}{12S^3} + \frac{\mathfrak{S}^6}{36S^5} + \dots$$

Encke's *Olbers*, p. 70, Note.

If we write  $\eta = \frac{\mathfrak{S}}{S^{\frac{2}{3}}}$ , the series becomes

$$k^2 = \frac{\mathfrak{S}^2}{S} \left\{ 1 + \frac{\eta^2}{12} + \frac{\eta^4}{36} + \dots \right\},$$

where the series within the brackets is  $\mu^2$ ;  $\log \mu$  being the quantity which is given with  $\eta$  as argument in Encke's table for the solution of Lambert's theorem.\*

In the series for  $k^2$  write  $k^2 + \delta(k^2)$  for  $k^2$ , and  $S + dS$  for  $S$ ; then

$$\begin{aligned} \delta(k^2) &= -\frac{\mathfrak{S}^2}{S^2} \left( 1 + \frac{\mathfrak{S}^2}{S^3} \right) \delta S \\ &= -\eta^2 S dS, \end{aligned} \tag{II}$$

omitting the terms of the expansion after the first, which may be done for approximate purposes.

Again the equation  $k^2 = u_1^2 + A^2$  gives

$$\delta(k^2) = 2u\delta u_1. \tag{III}$$

\* The expression  $k = \frac{\mathfrak{S}}{S^{\frac{2}{3}}} \mu$ , on which Encke's table is founded, is generally proved in the books by a trigonometrical transformation. The present method is preferable when once the series for  $k^2$  is regarded as known. This series occurs in Olbers' work, and it looks very much as if Encke's table had been originally formed in this way.

Combining I. II. III., we find

$$\delta u_1 = -\delta u_0 \frac{S\eta^2}{2u} \left( \frac{u+c}{rb^2} + \frac{u+c''}{r''b''^2} \right);$$

or if

$$\delta u_1 = -F \delta u_0,$$

$$F = \frac{S\eta^2}{2u} \left( \frac{u+c}{rb^2} + \frac{u+c''}{r''b''^2} \right);$$

and we have the following conclusions :—

- (1) The true value of  $u$  always lies between  $u_0$  and  $u_1$  ;
- (2)  $u = u_1 + (u_0 - u_1) \frac{F}{F+1}$  will be an approximate value of  $u$ .

The convenience of the process will be best illustrated by an example. That which I have selected is worked in different ways, as an example of the various methods, in Klinkerfues' *Theoretische Astronomie*, 1871, pp. 95-119. The number of successive hypotheses required to attain a solution is considerable, according to all the methods given.

I shall now give the computation of one hypothesis, and its improvement by means of the function  $F$ . The calculation of a second hypothesis, founded on the improved value, and the improvement of the value found in the second calculation are sufficient for accuracy to five places ; but as the second calculation is similar to the first, I give only its results. The further proceedings do not fall within the scope of this communication.

(Throughout the calculations,  $G. L.$  stands for "Gaussian logarithm"—i.e. addition or subtraction logarithm.)

Comet II. of 1813 :—

|                  |             |
|------------------|-------------|
| $t = 7.55002$    | April 1813. |
| $t' = 14.54694$  |             |
| $t'' = 21.59931$ |             |

|                                   |                                |
|-----------------------------------|--------------------------------|
| $\lambda = 271^{\circ} 16' 38''$  | $\beta = 29^{\circ} 2' 0''$    |
| $\lambda' = 266^{\circ} 27' 22''$ | $\beta' = 22^{\circ} 52' 18''$ |
| $\lambda'' = 256^{\circ} 48' 8''$ | $\beta'' = 9^{\circ} 53' 12''$ |

|                                 |                      |
|---------------------------------|----------------------|
| $\odot = 17^{\circ} 47' 41''$   | $\log R = 0.00091$   |
| $\odot' = 24^{\circ} 38' 45''$  | $\log R' = 0.00175$  |
| $\odot'' = 31^{\circ} 31' 25''$ | $\log R'' = 0.00260$ |

Whence we have for the Gaussian quantities

|                            |                     |
|----------------------------|---------------------|
| $\log M = 9.75799$         | $\log B = 9.98706$  |
| $G = 113^{\circ} 43' 57''$ | $\log B' = 9.86038$ |

|                              |                          |
|------------------------------|--------------------------|
| $\log g = 9\ 38029$          | $\log b = 9\ 75645$      |
| $H = 109^\circ\ 5'\ 49''$    | $\log b'' = 0\ 05028$    |
| $\zeta = 44^\circ\ 13'\ 9''$ | $g \cos \phi = 0\ 17147$ |
| $\log h = 9\ 81477$          | $c = 0\ 31365$           |
| $\log A = 9\ 22527$          | $c'' = 0\ 95443$         |

Assume for first hypothesis  $u_o = 0\ 3$  (Encke's preliminary rule gives  $0\ 2984$ ). Then we have in the usual way

|                                      |                                   |
|--------------------------------------|-----------------------------------|
| $u_o + c = 0\ 61365$                 | $u_o + c'' = 1\ 25443$            |
| $\log (u_o + c) = 9\ 78793$          | $\log (u_o + c'') = 0\ 09844$     |
| $\log \frac{1}{b} = 0\ 24355$        | $\log \frac{1}{b''} = 9\ 94972$   |
| <hr/>                                | <hr/>                             |
| $0\ 03148$                           | $0\ 04816$                        |
| $0\ 06296$                           | $0\ 09632$                        |
| $\log B^2 = 9\ 97412$                | $\log B''^2 = 9\ 72076$           |
| <hr/>                                | <hr/>                             |
| Diff. $0\ 08884$                     | Diff. $0\ 37556$                  |
| G. L. $0\ 34772$                     | G. L. $0\ 52820$                  |
| <hr/>                                | <hr/>                             |
| $\log r^2 = 0\ 32184$                | $\log r''^2 = 0\ 24896$           |
| $\log r = 0\ 16092$                  | $\log r'' = 0\ 12448$             |
| $\log r'' = 0\ 12448$                |                                   |
| <hr/>                                |                                   |
| Diff. $0\ 03644$                     |                                   |
| G. L. $0\ 31963$                     |                                   |
| <hr/>                                |                                   |
| $\log (r + r'') = 0\ 44411 = \log S$ |                                   |
| $\log S^{\frac{1}{2}} = 0\ 22206$    | $\log \kappa = 8\ 53661$          |
| $\log \kappa (t'' - t) = 9\ 68426$   | $\log (t'' - t) = 1\ 14765$       |
| <hr/>                                | <hr/>                             |
| $9\ 46220$                           | $9\ 68426$                        |
| $\log \mu = 0\ 00020$                | $\log S^{\frac{3}{2}} = 0\ 66618$ |
| <hr/>                                | <hr/>                             |
| $\log k = 9\ 46240$                  | $\log \eta = 9\ 01808$            |
| $\log k^2 = 8\ 92480$                | $\eta = 0\ 10435$                 |
| $\log A^2 = 8\ 45054$                | $\log \mu = 0\ 00020$             |
| <hr/>                                |                                   |
| Diff. $0\ 47426$                     |                                   |
| G. L. $0\ 29672$                     |                                   |
| <hr/>                                |                                   |
| $\log u_1^2 = 8\ 74726$              |                                   |
| $\log u_1 = 9\ 37363$                | $u_1 = 0\ 23639$                  |

And since  $u_0$  was 0.3,

$$u_0 - u_1 = 0.06361$$

$$\log(u_0 - u_1) = 8.80353.$$

We proceed to calculate,

$$F = \frac{S\eta^2}{2u} \left( \frac{u+c}{rb^2} + \frac{u+c''}{rb''^2} \right),$$

noticing that all the logs we require are already on the paper.

$$\log \frac{u+c}{b} = 0.03148$$

$$\log \frac{u+c''}{b''} = 0.04816$$

$$\log \frac{1}{b} = 0.24355$$

$$\log \frac{1}{b''} = 9.94972$$

$$0.27503$$

$$9.99788$$

$$\log r = 0.16092$$

$$\log r' = 0.12448$$

$$0.11411$$

$$9.87340$$

$$9.87340$$

$$\text{Diff. } 0.24071$$

$$\text{G. L. } 0.43785$$

$$0.31125$$

$$\log \eta^2 = 8.03616$$

$$\log u_1 = 9.37363$$

$$\log S = 0.44411$$

$$\log z = 0.30103$$

$$8.79152$$

$$9.67466$$

$$9.67466$$

$$9.11686 = \log F$$

$$\text{G. L. } 0.05341 = \log(F+1)$$

$$\log \frac{F}{F+1} = 9.06345$$

$$u_1 = 0.23639$$

$$\log(u_0 - u_1) = 8.80353$$

$$0.007362$$

$$7.86698$$

$$U_1 = 0.24375$$

$$(u_0 - u_1) \frac{F}{F+1} = 0.0073618$$

The ultimate value of  $u$  on the assumed data is  $u = 0.24384$ , according to my repeated verifications. (Klinkerfues states that it is 0.24389; this difference may possibly arise from some further corrections being admitted in the data). Consequently by this simple calculation, based on the results of one

ordinary computation, we have arrived within one unit in the fourth place: a far more expeditious approximation than that given by any process I have found in the books.

It will always be necessary to make a second computation for the purpose of a check. With reference to this we may observe that careful five-place work is well repaid in the second computation, though not in the first. But for the second value of  $F$  three places are sufficient, though we shall obtain five for illustration; in fact, the influence of the exactness of  $F$  in the second computation is so small that the result in this example is the same whether we recompute  $F$  or not. Proceeding in exactly the same manner as before, but employing  $U_1 = 0.24375$  instead of  $u_0 = 0.3$ , we find

$$u_2 = 0.24385$$

and therefore

$$U_1 - u_2 = -0.00010$$

Recomputing  $F$ , we find

$$\begin{aligned} \log. F &= 9.12483 \\ G. L. = \log (F + 1) &= 0.05435 \\ \hline 9.07048 &= \log \frac{F}{F + 1} \\ \text{Num. log } (U_1 - u_2) &= 6.00000 \\ \hline 5.07048 \end{aligned}$$

and therefore

$$(U_1 - u_2) \frac{F}{F + 1} = -0.000012$$

and to five places

$$U_2 = 0.24384.$$

A recalculation gave for  $u_3$  the same value to the last place; consequently the performance of two complete calculations in the manner indicated above has here been sufficient for five-place purposes.

The successive results are now placed together.

|                 | Corrections given<br>by present method |
|-----------------|----------------------------------------|
| $u_0 = 0.3$     |                                        |
| $u_1 = 0.23639$ | + 0.00736                              |
| $U_1 = 0.24375$ |                                        |
| $u_2 = 0.24385$ | - 0.00001                              |
| $U_2 = 0.24384$ |                                        |

I may notice briefly that a useful check is given by the first

deduction from our investigation (p. 356), viz. that the true value of  $u$  lies between  $u_0$  and  $u_1$ , the initial and final values of one of the ordinary calculations, as exhibited at p. 359. For instance, in the present case, the final number ( $U_2$ ) lies between both pairs,  $u_0-u_1$ , and  $U_1-u_2$ .

But the value of  $u$  given by Klinkerfues (p. 97) as that which would be obtained, "setzen wir die Versuche bis zur grössten Schärfe fort," is 0.24389, which falls beyond the limits  $U_1-u_2$  by four units of the last place. Here certainly either my theory was wrong, or my computation of  $U_1-u_2$  was wrong, or Klinkerfues was wrong. I believe, after a good deal of troublesome verification, that Klinkerfues' value is wrong upon the data assumed.

I think there can be no question that the present method gets over one of the most troublesome and unsatisfactory parts of the computation of parabolic orbits better than any method in common use.

Observations of the Companion of a Canis Majoris with the Transit Circle of the U.S. Naval Observatory. By J. R. Eastman, Esq.

| Date.    | Object.                | Seconds<br>of<br>Transit.<br>s | Micrometer<br>Readings. | $\Delta\alpha$<br>s | $\Delta\delta$<br>r | $p$<br>o | $s$<br>" |
|----------|------------------------|--------------------------------|-------------------------|---------------------|---------------------|----------|----------|
| March 20 | $\alpha$ Canis Majoris | 11.362                         | 33.070                  | 0.622               | 0.502               | 49.3     | 11.8     |
|          | Companion              | 11.984                         | 32.568                  |                     |                     |          |          |
| „ 22     | $\alpha$ Canis Majoris | 10.965                         | 33.235                  | 0.631               | 0.415               | 55.0     | 11.1     |
|          | Companion              | 11.596                         | 32.820                  |                     |                     |          |          |
| „ 23     | $\alpha$ Canis Majoris | 10.910                         | 33.235                  | 0.644               | 0.420               | 55.2     | 11.3     |
|          | Companion              | 11.554                         | 32.815                  |                     |                     |          |          |

One revolution of the micrometer screw = 15".312.  
The transit of  $\alpha$  Canis Majoris was observed over six threads and the companion over five threads. On March 20 both stars were unusually steady, and the observations of the difference of declination were much better than on the other nights.  
Of course these observations have but little value in fixing the position of the companion, and are given only to show that it may be observed on a meridian instrument of 8.5 inches aperture.

Naval Observatory, Washington,  
March 25, 1878.

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